Engineering field theory with applications

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Introduction

1.1 Introduction

Engineering Field Theory with Applications is intended to give a unified mathematical treatment to several disciplines of engineering. This unified field theory is common to electric, magnetic and gravitational fields to name a few; this commonality aids our understanding. First, however, what is a field? Most of us experience fields regularly and may not realize it. For example, as you read this page it is in an electric field and you are sitting (or lying down) in a thermal field. Then, all of us are in a gravitational field. These fields are characterized by the fact that each point in space has its own (field) value. So we say a field is a spatial arrangement – imaginary or real – of any parameter. The space may be one-, two-, or three-dimensional depending on the quantity. A temperature field is a one-, two-, or three-dimensional arrangement of points at which we determine a temperature. All the points together constitute a field. The kitchen oven is an example of a three-dimensional temperature field. A gravitational field is a three-dimensional arrangement of forces of gravity. A spacecraft is subjected to these forces as it flies through the field. The gravitational force is exerted by the planet on the craft. Each spatial point can be represented by a different gravitational constant; there are an infinite number of values. These fields exist individually or coexist. In addition, there are fields which interact. For example, in electromagnetics, electric and magnetic fields interact. The electric field gives rise to the magnetic field and the magnetic field gives rise to the electric field. From this we get wave propagation. In fluids, temperature and velocity fields interact. An increase in the temperature of a fluid causes atoms and electrons to be agitated at a higher velocity. The fluid's velocity increases. The inverse is true: an increase (or decrease) in the velocity field of a fluid will cause an increase (or decrease) in the temperature field.

Since there is this interaction and coexistence between engineering fields, it makes sense to look at their bases to see what they have in common. The common language of engineering is mathematics. Are the equations used in the fields the same, similar, or completely different? It turns out that the answer is *yes*. Some equations are the same, some equations are similar and some equations are different. We will point these out as we proceed on our odyssey through the book. This technique of examining sameness, similarities and differences will hopefully strengthen our study. This trend or evolution of pedagogy is actually a broadening of what is already done in most curricula. For example, electricity and magnetism are taught together in electrical engineering courses today when at first they were not even connected by any unifying laws.

Today the engineering graduate faces the demands of society in a variety of fields. The engineer designing an amplifier must be concerned with more than the electronic design. In addition, he must be concerned with heat transfer and space optimization. Gone are the days when the electrical engineers, mechanical engineers and industrial engineers would operate in isolated cloisters. As another example take the automotive engineer. What once was the domain of the mechanical engineer now includes complex problems such as air pollution, fuel efficiency, electronic ignitions and on-board computers. More could be said about areas such as space exploration, microelectronics and electrical power generation. Suffice it to say that today's engineer must be versed in more than one field. Let's now look at the common language of engineering.

1.2 Mathematical basis

Mathematics is the language of engineering. Mathematical equations or expressions describe certain behavior in engineering. There are several mathematical expressions common to the engineering disciplines we study in this book. The first is Poisson's equation.

$$\nabla^2 f = \text{constant} \tag{1.1}$$

where f is some spatial function.

If the right side of equation (1.1) is zero, we obtain Laplace's equation, or

$$\nabla^2 f = 0 \tag{1.2}$$

The solutions of equations (1.1) and (1.2) are the potential functions. From these we can obtain the field intensity vectors. This is obtained by taking the derivative of the potential. Once we have the field intensity, we are able to determine the flux density.

Another important mathematical principle we will utilize in this book is Gauss' law. Gauss' law states, 'the flux emanating from a closed surface is the result of the charge enclosed by that surface'.

$$\Phi \equiv Q = \oint \mathbf{D} \cdot \mathrm{d}\mathbf{A} \tag{1.3}$$

where Q is some enclosed source, **D** is a flux density and **A** is the area which encloses the source. It is a most useful law when the problem has symmetry.

That is, for symmetric sources the flux density, **D**, in equation (1.3) is moved outside the integral. This simplifies the integration. Once we obtain the density, we can find the field intensity and potential. Equations (1.1) (or (1.2)) and (1.3) are two different methods of solving field problems. Equation (1.1) gives us the potential function while equation (1.3) gives us the density function. The potential and density functions are related and their relationship will be shown in later chapters. Naturally both methods should give the same results.

A third important equation we will utilize is Stokes' theorem. Stokes' theorem relates a surface integral to a line integral. The surface in this case is an open surface as opposed to the Gaussian surface in equation (1.3) above, which is a closed surface. Stokes' theorem is written

$$\oint \mathbf{F} \cdot \mathrm{d}\mathbf{l} = \iint \nabla \times \mathbf{F} \cdot \mathrm{d}\mathbf{A} \tag{1.4}$$

for any vector **F**. The **dl** path in equation (1.4) encloses the open surface indicated on the right-hand side.

A fourth important equation is the divergence theorem. While Stokes' theorem relates a closed line integral to an open area integral, the divergence theorem relates a closed surface to a volume. This is written

$$\oint \mathbf{D} \cdot \mathrm{d}\mathbf{A} = \iiint \nabla \cdot \mathbf{D}\, \mathrm{d}v \tag{1.5}$$

where v is volume. Thus, for any vector density, **D**, the divergence theorem says, if we add all the densities which are at an enclosing surface, the result will equal the divergence of **D** throughout the volume. The surface on the left of equation (1.5) encloses the volume on the right. This is the integral form of Maxwell's first equation. More will be said about that later.

1.3 Gradient

The last mathematical operation we mention in this introduction is the gradient. Gradient is the maximum rate of change of potential with position. The gradient is a vector quantity. For conservative fields the gradient of the potential equals minus the field intensity.

In electrostatics we write

$$\mathbf{E} = -\nabla V \quad \text{volts/meter}$$

where $\mathbf{E}$ is the electric field intensity.

In magnetostatics we write

$$\mathbf{H} = -\nabla V_{\mathrm{m}} \quad \text{amps/meter}$$

where $\mathbf{H}$ is the magnetic field intensity and V_{m} is magnetic scalar potential.

In gravitational fields we write

$$\mathbf{g} = -\nabla V_g \quad \text{Newtons/km or meters/second}^2$$

where $\mathbf{g}$ is the gravitational field intensity and V_{g} is gravitational potential.

In fluids we write

$$\mathbf{u} = -\nabla V_{\mathrm{f}} \quad \text{meters/second}$$

where $\mathbf{u}$ is velocity and V_{f} is fluid potential.

In heat transfer by conduction we write

$$\mathbf{q} = -k\nabla T \quad \text{watts/meter}^2$$

where $\mathbf{q}$ is heat flux density, k is thermal conductivity and T is temperature.

In electrical conduction we write

$$\mathbf{J} = -\sigma\nabla V \quad \text{amps/meter}^2$$

where $\mathbf{J}$ is a current density vector and σ is conductivity.

In fluid flow through permeable media we write

$$\mathbf{u} = -k\nabla p \quad \text{meters/second}$$

where $\mathbf{u}$ is velocity, k is soil permeability and p is pressure.

In diffusion we write

$$\mathbf{D}_{\mathrm{d}} = -k_{\mathrm{d}}\nabla N \quad \text{particles/meter}^2\ \text{second}$$

where $\mathbf{D}_\mathrm{d}$ is particle flux density, k_d is the diffusion coefficient and N is particles per unit volume.

Finally, in acoustics we write

$$\mathbf{u} = -\left(\frac{1}{j\omega\rho_0}\right)\nabla p \quad \text{meters/second}$$

where $\mathbf{u}$ is velocity of the sound wave, ω is radian frequency, ρ_0 is density and p is pressure.

We have attempted to show a pattern in this introduction of concepts involving fields and fluxes. It is obvious that the fields and fluxes are different quantities with different units. The mathematical exercises are the same.

Problems

1. List five engineering occupations which require more than one engineering field. For each occupation list the different engineering fields required.

2. List three countries other than North American countries which can utilize engineers with a broad background – preferable to highly specialized engineers – and specify the desirable engineering backgrounds.